

Bitcoin Allocation

HKU 2026 SUMMER CAMP

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Our team

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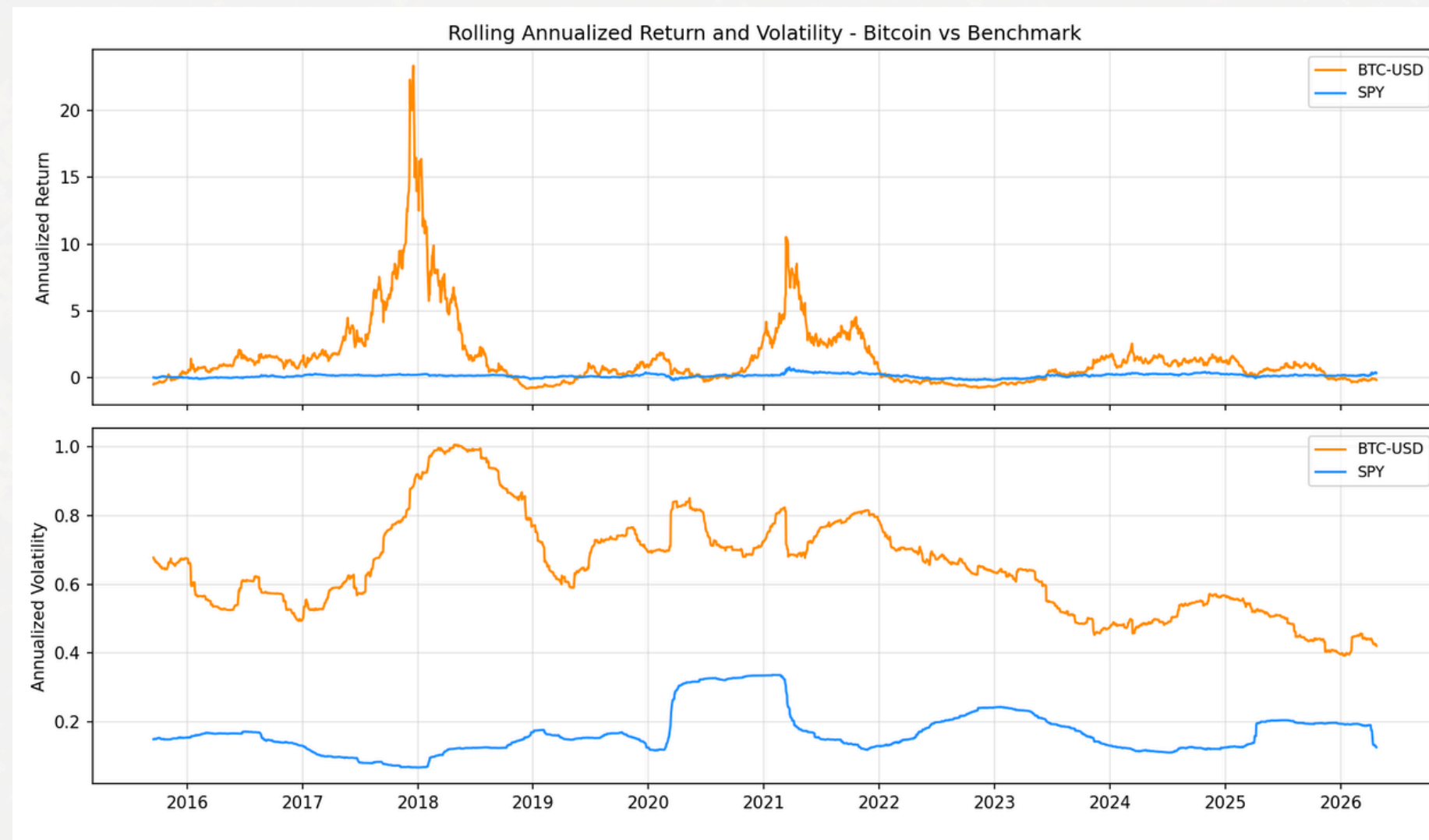
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Bitcoin mobilizes household excess cash

- Households tend to be cautious with risky assets, leaving excess cash idle.
- Bitcoin is a high-return, high-risk asset.
 - If right-sized, it opens a window for idle cash to mobilizes.



Problem Statement

How much Bitcoin should a household hold?

Our Client Profile

Mass Affluent Professional household:

- High human capital and discretionary income
- Demand for allocating complicated assets
- Goal: Long-term capital appreciation

Assumptions & Constraints:

- Target Annualized Volatility: 12% - 15%
- Liquidity → Expected Shortfall Cap

We simulated a portfolio including 4 ETFs and Bitcoin.

Asset	What it represents	Ticker
SPY	US large-cap equities	SPY
VEU	International equities	VEU
TLT	Long-duration US bonds	TLT
GLD	Gold / inflation hedge	GLD
BTC	Bitcoin	BTC-USD

- Aligned BTC (24/7) to U.S. trading days to match other assets.
- Used dynamic risk-free rate from the 3-month T-Bill (IRX)

Understanding Assets: Intuitions

Why equities > bonds?

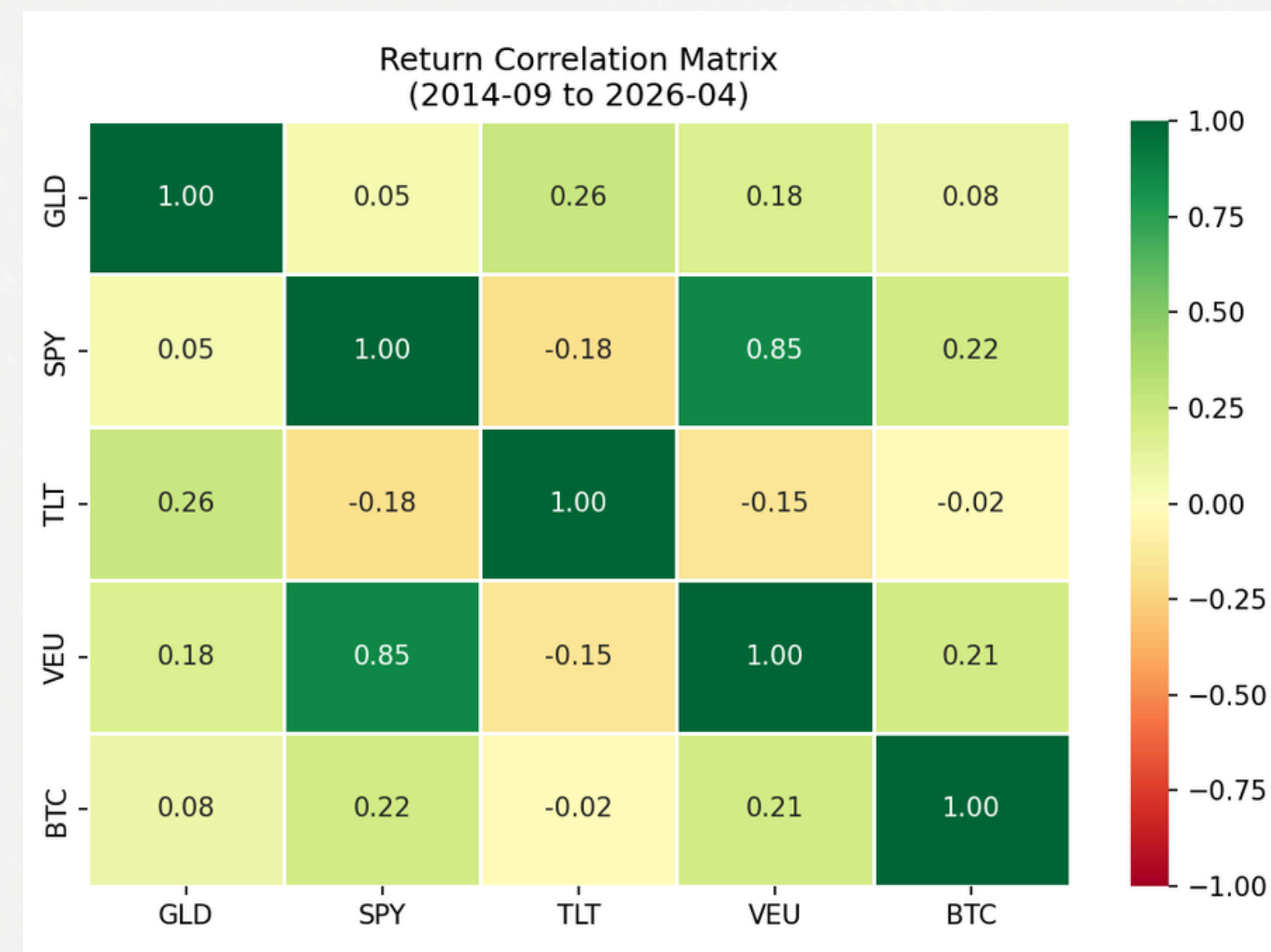
- **Equity risk premium** to bear the cash-flow and liquidation risk.

Why does BTC earn a return?

- **Digital Scarcity:** Fixed supply (21M coins) plus growing demand forces the price up.
- **Network effect / Metcalfe's Law:** The value of the network grows super-linearly in the number of users. Early adopters are compensated for bearing adoption risk.
- **Technology risk premium:** Positive expected return as compensation for the risk of returning to 0.

Bitcoin look risky alone, but its low correlation made it a powerful diversifier

Asset	Ann. Return	Ann. Vol	Ann. Sharpe	Max Drawdown
GLD	13.4%	15.9%	0.70	-22%
SPY	15.3%	17.6%	0.74	-34%
TLT	1.6%	14.9%	-0.02	-48%
VEU	8.7%	17.2%	0.39	-35%
BTC-USD	94.4%	66.5%	1.36	-83%



Bitcoin Characteristics & Statistics

Non-normality
Fat-tail

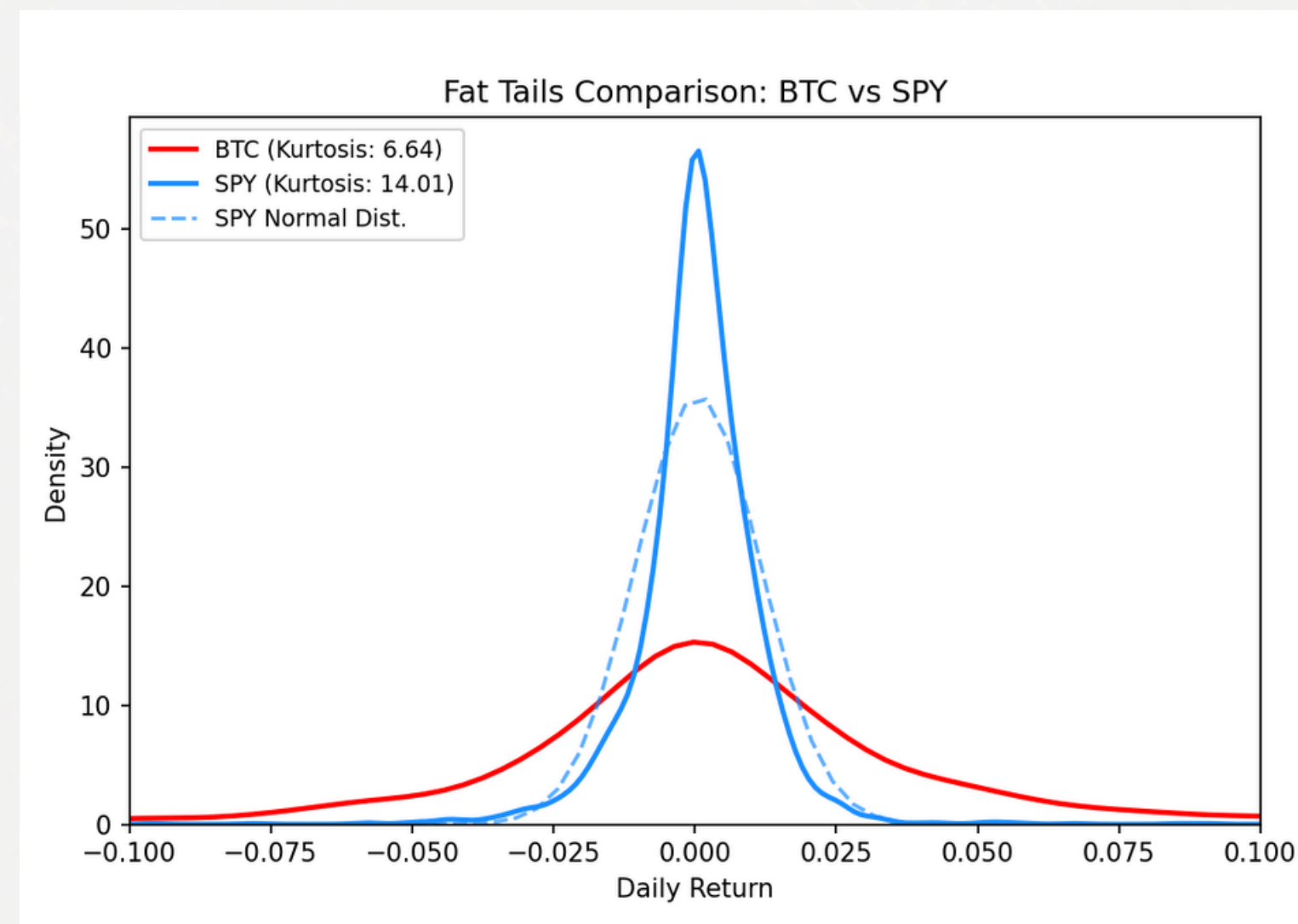
Time-varying
Statistics

Market Beta
Structural Shift

Bitcoin Statistics

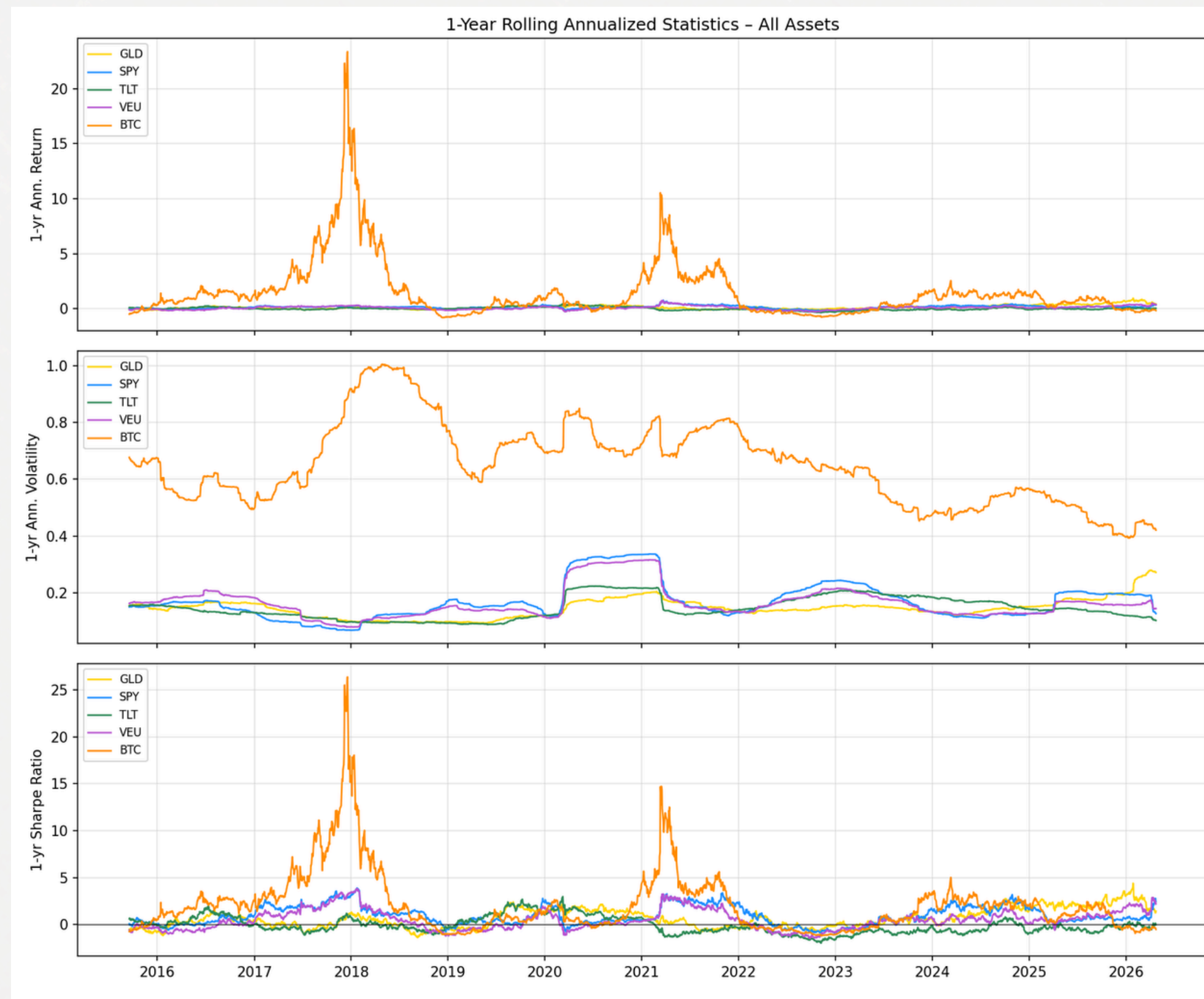
1 The Fat-Tail Reality

Metric	SPY (US Equity)	BTC (Bitcoin)
Ann. Return	15.29%	94.41%
Ann. Volatility	17.62%	66.46%
Max Drawdown	-33.72%	-83.04%
Daily 95% CVaR	-2.67%	-9.56%
P(Drop > 5%)	0.21%	7.37%



Bitcoin Statistics

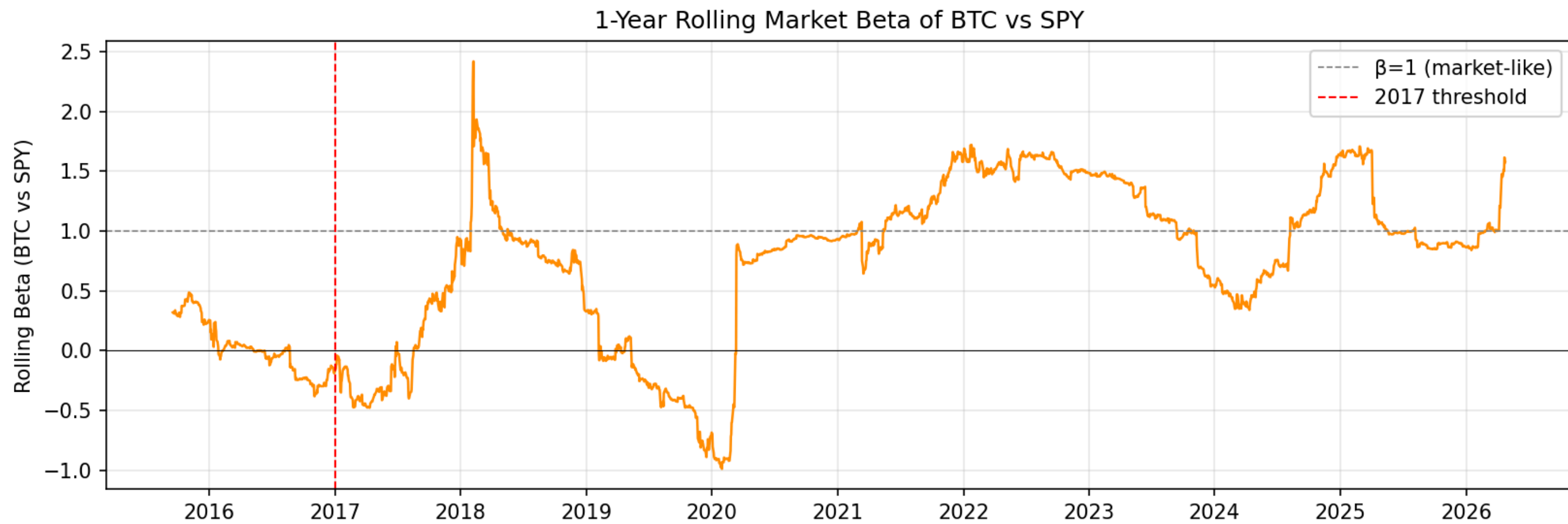
2 Time-varying Statistics



Bitcoin Statistics

3 Market Beta Structural Shift

Period	Beta	Alpha (ann.)	R-squared
Full sample	0.85	72.3%	0.050
Pre-2017	0.01	65.3%	~0.000
Post-2017	0.97	73.6%	0.068



We used MVO and Black-Litterman model to optimize asset allocation

Phase 1

Mean-Variance Optimization (MVO)

Phase 2

Black-Litterman Algorithm

MVO can find the best trade-off between expected return and risk

- Identifies the asset weights that deliver the best risk-adjusted return.
- We designed 6 optimization methods, depending on the end goal
 - maximize Sharpe ratio
 - maximize return
 - Volatility-constrained
 - 10% p.a.
 - 15% p.a.
 - Expected Shortfall (ES) \leq 5%
 - ES \leq 25%
 - BTC-USD cap at 10%

Assumption 1:

Gaussian Distribution

Reality:

Fat Tails

Result:

Left-tail risk ignored, High MaxDD

Approach:

Introduce the ES constraint

Mathematical equations for MVO - max Sharpe Ratio

Setup: w = weights, μ = expected returns, Σ = covariance matrix, $\sum w_i = 1$

Portfolio return: $\mathbb{E}[R_p] = w^T \mu$

Portfolio risk (variance): $\sigma_p^2 = w^T \Sigma w$

Optimization: $\min_w \frac{1}{2} w^T \Sigma w$ s.t. $w^T \mu = \mu^*$, $\sum w_i = 1$

Sharpe ratio: $\text{Sharpe} = \frac{w^T \mu - r_f}{\sqrt{w^T \Sigma w}}$

Mathematical equations for MVO - volatility cap

$$\sigma_{\max} = 10\% \text{ p.a.: } \max_w w^T \mu \quad \text{s.t.} \quad \sqrt{w^T \Sigma w} \leq 0.10$$

$$\sigma_{\max} = 15\% \text{ p.a.: } \max_w w^T \mu \quad \text{s.t.} \quad \sqrt{w^T \Sigma w} \leq 0.15$$

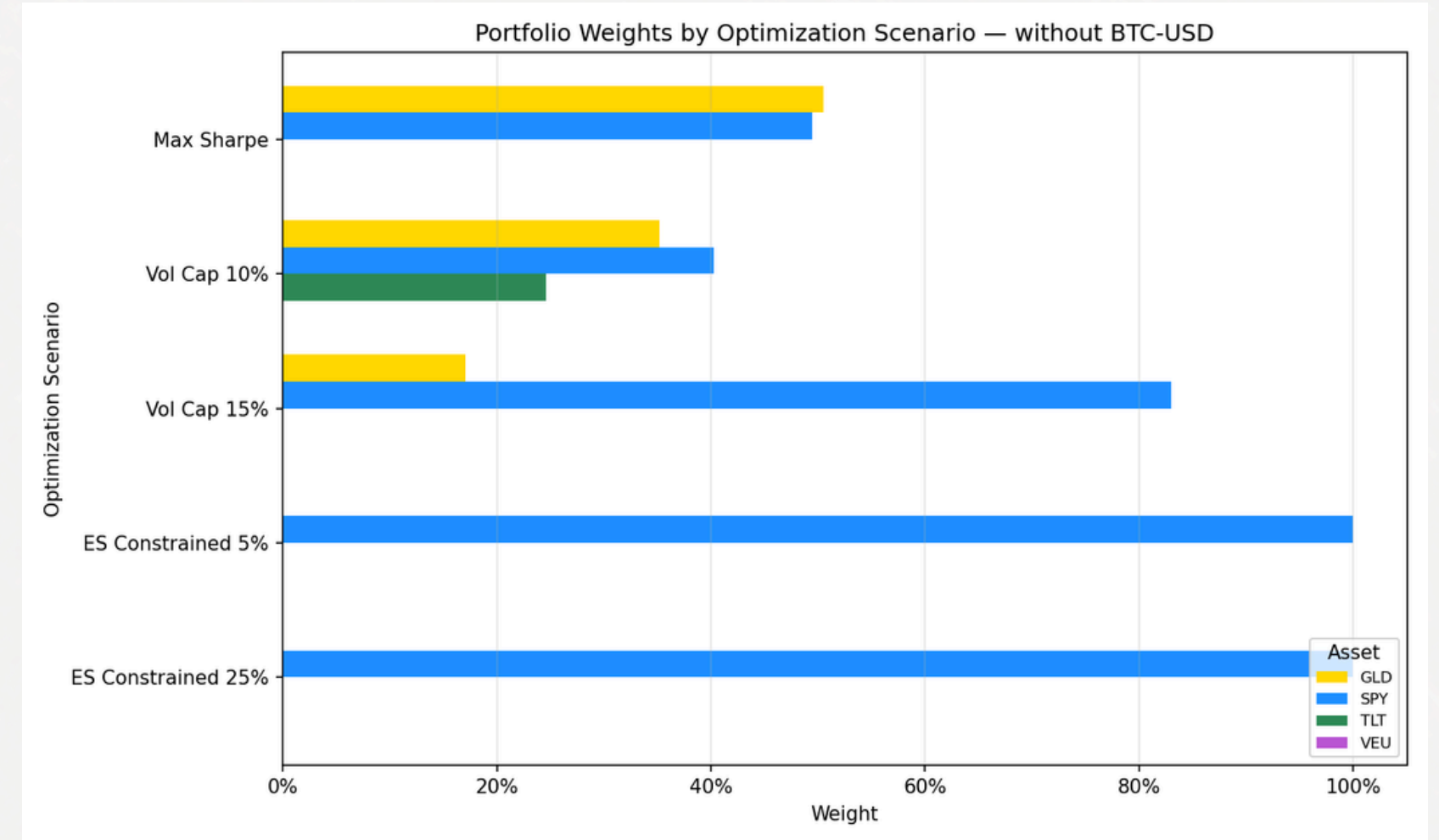
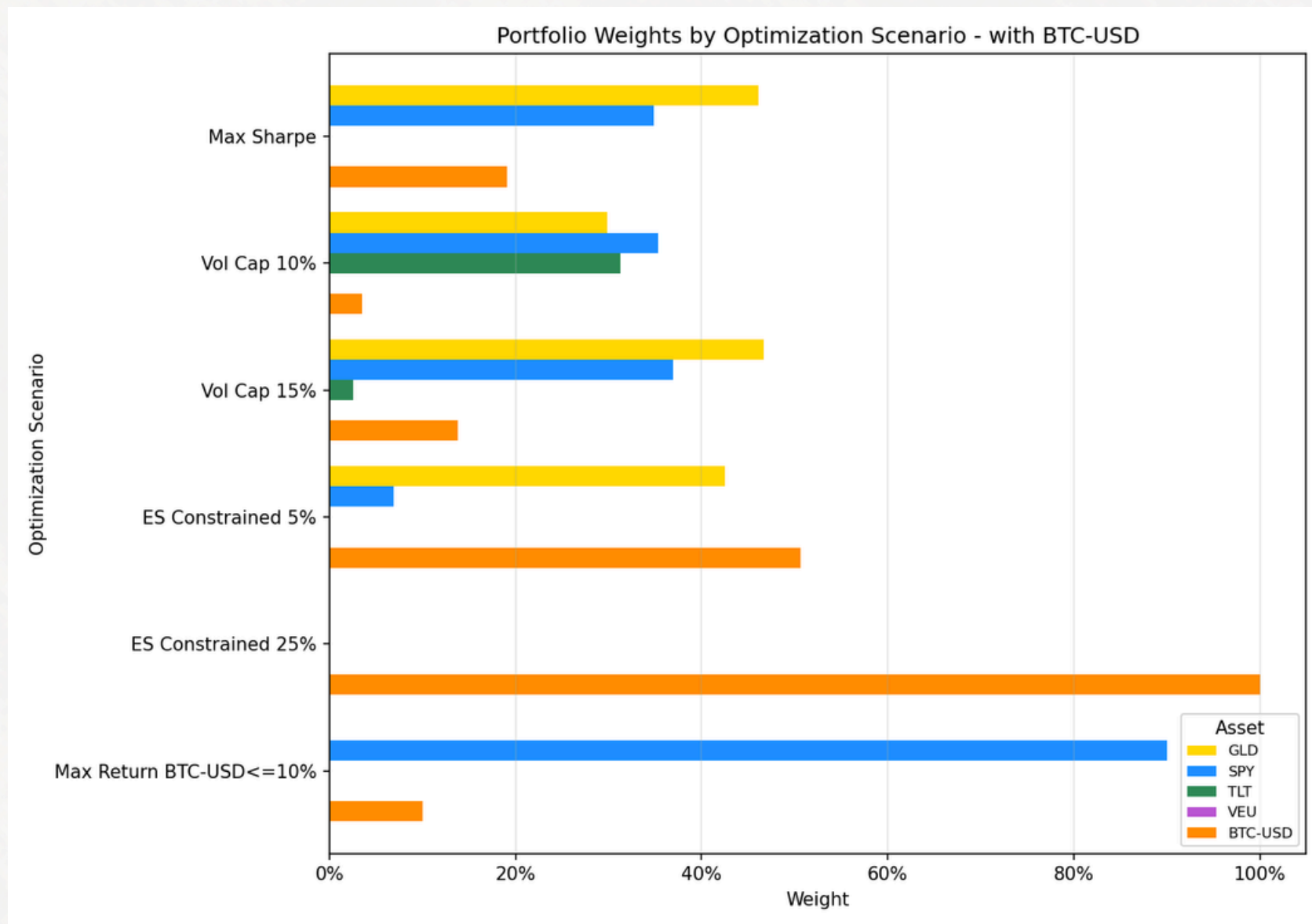
Mathematical equations for MVO - Expected Shortfall (ES) $\leq 5\%$

$$\max_w w^T \mu \quad \text{s.t.} \quad \text{ES}_{0.05}(w) \leq 0.05, \quad \sum w_i = 1$$

Mathematical equations for MVO - BTC-USD cap at 10%

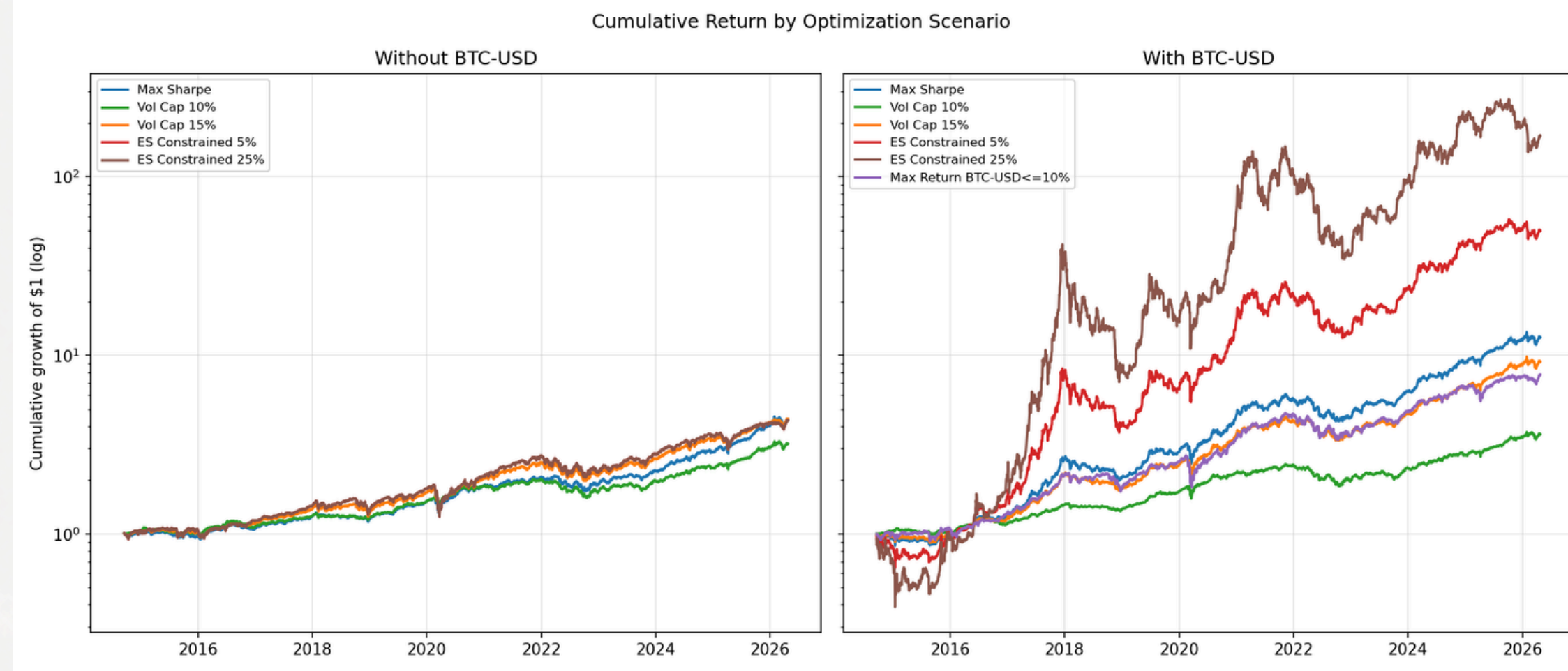
$$\max_w w^T \mu \quad \text{s.t.} \quad w_{\text{BTC}} \leq 0.10, \quad \sum w_i = 1$$

Bitcoin doesn't just add return — it displaces less efficient assets across every scenario

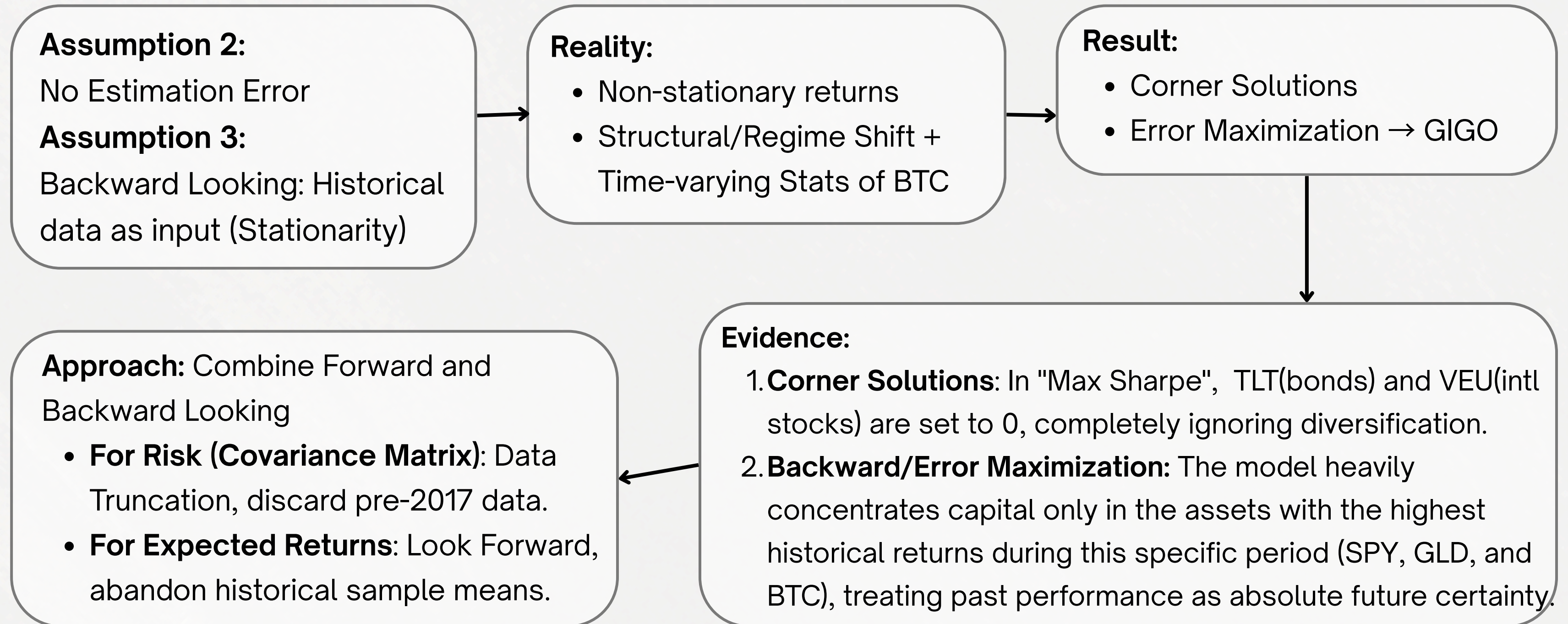


Adding Bitcoin at the optimal weight has achieved our objective

Scenario	Ann.Ret (w/o)	Ann.Ret (w/)	Vol (w/o)	Vol (w/)	Sharpe (w/o)	Sharpe (w/)	MaxDD (w/o)	MaxDD (w/)
Max Sharpe	13.4%	23.4%	12.2%	17.5%	0.81	1.13	-20.2%	-29.3%
Vol Cap 10%	10.5%	11.6%	10.0%	10.0%	0.69	0.80	-21.3%	-24.9%
Vol Cap 15%	14.0%	20.3%	15.0%	15.0%	0.69	1.12	-29.0%	-26.2%
ES Constrained 5%	14.2%	40.0%	17.6%	35.1%	0.60	1.04	-33.7%	-56.2%
ES Constrained 25%	14.2%	66.6%	17.6%	66.5%	0.60	0.95	-33.7%	-83.0%
Max Return BTC<=10%	--	19.5%	--	18.5%	--	0.86	--	-33.5%



Why MVO is limited?

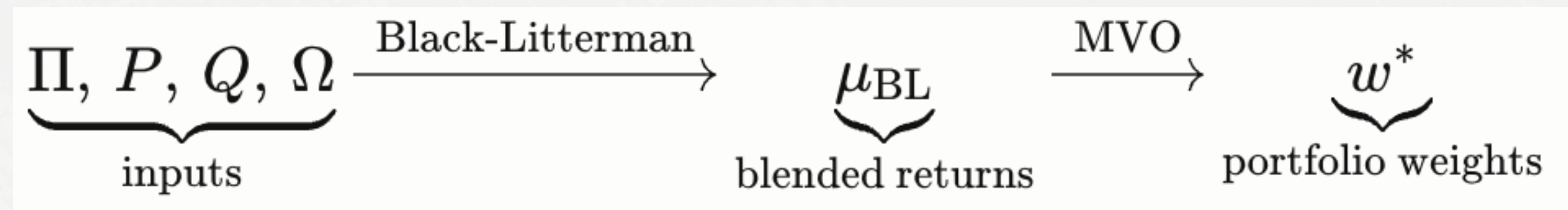


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Black-Litterman model combines the market's prior with investors' own views

**ASSET ALLOCATION:
COMBINING INVESTOR VIEWS
WITH MARKET EQUILIBRIUM**

FISCHER BLACK AND ROBERT LITTERMAN

Black-Litterman stabilizes expected returns before MVO finds the optimal weights



- We designed 3 views
 - Bitcoin will outperform international equities (VEU) by 15% per year.
 - The S&P 500 (SPY) will return 10% per year.
 - Gold (GLD) will outperform long-term Treasuries (TLT) by 3% per year.

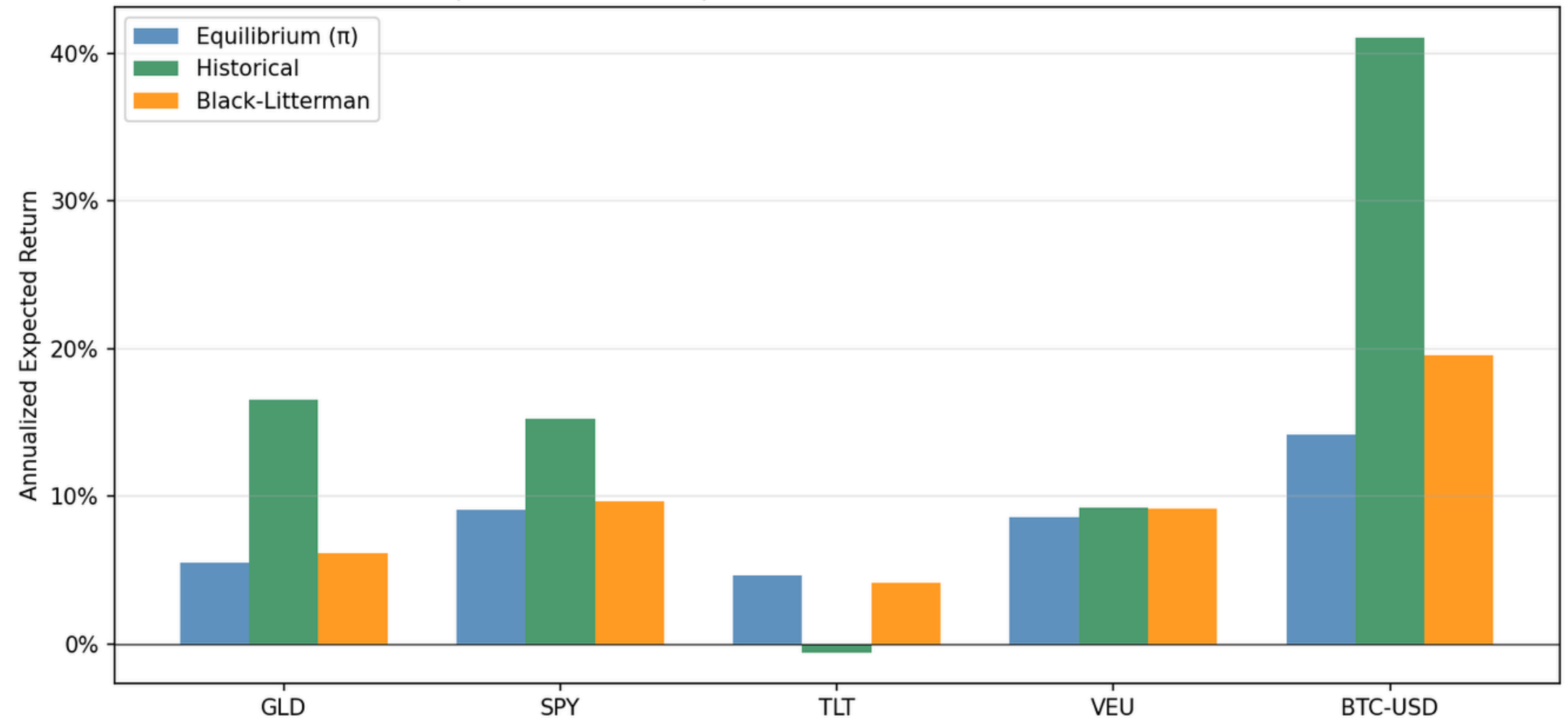
Mathematical equations for Black-Litterman Model

$$\mu_{\text{BL}} = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1} [(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$$

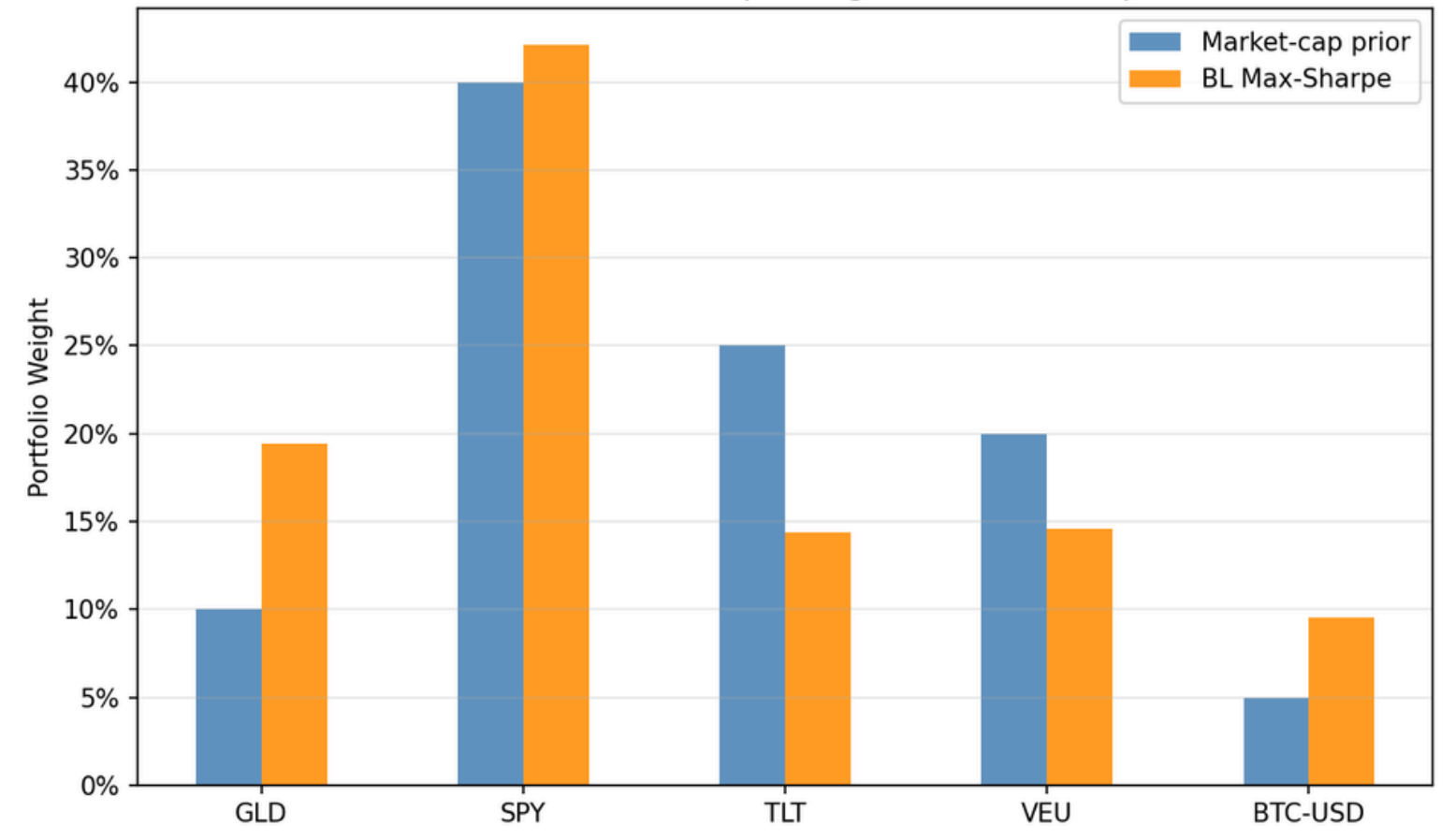
- μ_{BL} posterior expected return
- τ confidence in the equilibrium prior
- Σ covariance matrix
- P link matrix
- Ω uncertainty in the views
- Π implied equilibrium prior returns
- Q the view's expected return

Black-Litterman is pulling those extremes back to reality

Expected Returns: Equilibrium vs Historical vs Black-Litterman



Black-Litterman Max-Sharpe Weights vs Market-Cap Prior



Recommendation and Stance

Risk-Managed Pro-Crypto: hold 5-10% BTC.

Asset	Weight	Rationale
SPY	~40%	US equity core
VEU	~20%	International diversification
GLD	~15%	Inflation hedge, low BTC correlation
TLT	~10%	Duration / deflation hedge
BTC	~10%	Growth optionality, diversifier

Limitations & Future Enhancements

1. **BL input:** Static expected return → Dynamic expected return based on macro-driven signals
2. **Adopt Dynamic/Non-stationary Covariance:** EWMA or Ledoit-Wolf shrinkage matrix to capture dynamic correlations.
3. **Higher-moment optimization.**



Thank You

Zhongyi Guo, Yiran Feng